

Rigid foldability of the augmented square twist

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keywords: rigid folding, square twist, folding matrices

Abstract

The square twist is an interesting origami maneuver that has been used in origami art (e.g., Kawasaki’s Rose) and in origami science. The standard way to fold it, where the “inner diamond” of the twist is all mountain creases (see Figure 1(a)) is well-known to be not rigidly foldable, meaning that the crease pattern, with this mountain-valley (MV) assignment, cannot be folded flat without bending the faces of the crease pattern to get there. (See Hull (2013) for one proof of this.) In fact, this lack of rigid foldability has been used by researchers to study the mechanics of bistability in origami Silverbers et al. (2015).

If we add a crease to the square twist crease pattern, will it then be rigidly foldable? One possibility of augmenting the square twist crease pattern is shown in Figure 1(b), where a crease has been added across the inner diamond.

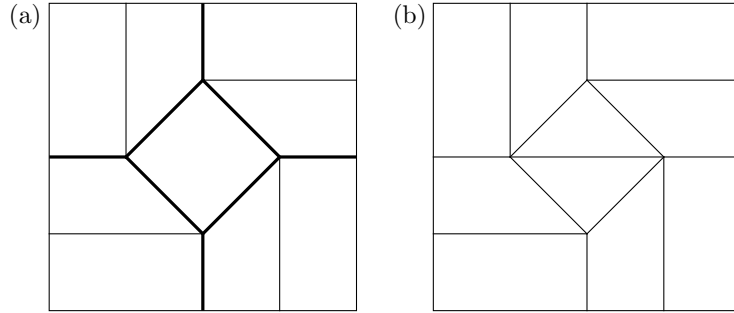


Figure 1: (a) The square twist; bold creases are mountains. (b) The augmented square twist.

In this paper we prove that the augmented square twist crease pattern in Figure 1(b) is rigidly foldable as a one degree-of-freedom origami mechanism. Specifically, there exists a rigid folding from the unfolded state to the flat-folded state where the creases from the original square twist use the standard MV assignment (Figure 1(a)). This rigid folding is non-trivial, requiring some creases to become mountains before they revert to valley creases in the flat-folded state. An illustration of this rigid folding is shown in Figure 2.

Furthermore, we prove that the augmented square twist crease pattern has only two modes, by which we mean MV assignments that allow the crease pattern to rigidly fold from the unfolded to a flat-folded state of the square twist. (We do not count modes where all the mountains and valleys are reversed as being different.) The first mode folds the crease pattern into the

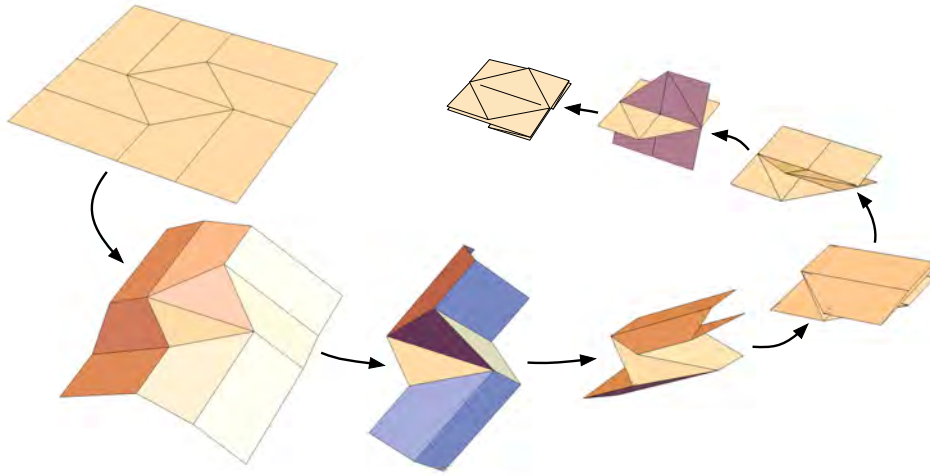


Figure 2: The augmented square twist rigidly folding into the standard square twist.

standard square twist (Figure 2). The second mode rigidly folds the crease pattern into the iso-area square twist, as depicted in Figure 3. The proofs of these claims rely on calculation of the configuration space of fold angles for this crease pattern, which is done using the matrix model for rigid folding.

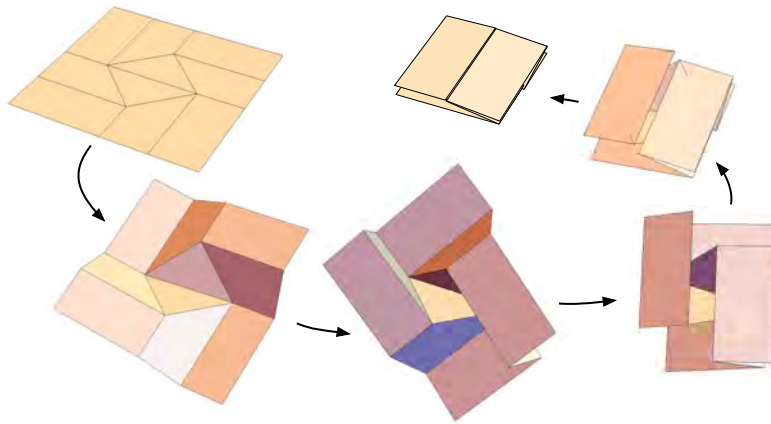


Figure 3: The augmented square twist rigidly folding into the iso-area square twist.

References

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